ROTATION OF TETRAHEDRAL NUCLEI IN THE CRANKING MODEL

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The three-dimensional cranking model is used to investigate the microscopic aspects
of the rotation of nuclei with the tetrahedral symmetry. Two classes of rotation axes
are studied corresponding to two different discrete symmetries of the rotating hamil-
tonian. Self-consistent Hartree-Fock-Bogoliubov calculations show that the tetrahedral
minimum remains remarkably stable until the first single-particle crossing.

1. Introduction

The prediction that atomic nuclei can possess stable low-lying configurations with
the tetrahedral symmetry has been confirmed by several independent calculations
using a variety of nuclear mean-fields[1,2,3,4]. However, no experimental evidence has
been reported so far, which can be put down to the lack of understanding of the
excitation mechanisms in such exotic systems. We investigate here the collective
rotation, since the deep minima and large barriers reported e.g. in Ref.[2] suggest
tetrahedral nuclei may be amenable to sustain rotational bands. This article is a
sequel to previous works where the structure and moments of inertia of tetrahedral
collective bands were analyzed[5], and the consequences of the collective rotation
in terms of symmetries and quantum numbers were discussed[6]. To complement
these studies, we wish to focus here on the stability of the tetrahedral minimum
as a function of the rotational frequency, which is one of the pre-requisites to an
eventual observation of rotational bands.

Today there exists several articles in the literature about the nuclear tetrahedral
symmetry (associated to the point group of symmetry T_d^D) and we refer the reader
interested in a general introduction to this subject to e.g. Ref.[7]. For the purpose
of the present study, it suffices to recall that a nucleus with a non-axial α_{32} oc-
tupole deformation (in the standard expansion of the nuclear radius on the basis
of spherical harmonics, with deformation parameters $\alpha_{\lambda\mu}$) has all the symmetries

of a regular tetrahedron. Although other realizations of the group T_d^D, involving higher-order multipole terms with $\lambda \geq 7$, also appear to generate stable minima in the potential energy landscape[8], they will not be considered in this article.

The cranking model was the subject of several comprehensive review articles and we refer to Refs.[9,10] for a general discussion of its main features. We just recall here how the model should be adapted in the 3-dimensional case: the rotation is described with the help of 3 Lagrange multipliers $\vec{\omega} \equiv (\omega_x, \omega_y, \omega_z)$ which are interpreted as the classical rotational frequencies along the x-, y- and z-axis of the body-fixed frame respectively. Equivalently, one can choose the spherical representation in which the rotational frequency vector $\vec{\omega}$ is parameterized by $(\omega, \theta, \varphi)$.

It is known that a collective rotation of a nucleus with prolate or oblate deformation takes place about an axis perpendicular to the symmetry axis. However, in the case of a nuclear shape with the T_d^D symmetry, the quadrupole deformation is equal to zero, and consequently no simple criterion exists to determine the sorientation of the rotation axis.

A possible procedure to find optimum rotation axis consists in computing the total energy as a function of the orientation of the rotation axis, characterized by the two angles (θ, φ), for different rotational frequencies ω, as was done in Refs.[5,6]. One thus obtains two-dimensional maps whose minima signal the energetically-favored axes of rotation. These calculations were performed using a macroscopic-microscopic technique, in which the total energy is the sum of a liquid-drop contribution parameterized as in Ref.[11] and shell-correction extracted from a Woods-Saxon potential with the form defined in Ref.[12]. The results[5,6] suggest that at low rotational frequencies (up to $\omega \sim 0.3$ MeV/$\hbar$), no particular axis of rotation is favored, while at higher frequencies, several well-defined minima emerge. However, these calculations assumed a fixed tetrahedral deformation and did not include pairing correlations: therefore, in the present study we want to see whether the tetrahedral minimum survives the increase of angular momentum.

2. Stability of the tetrahedral minimum

In order to study the stability of the tetrahedral minimum and to treat on the same footing the response of the nuclear system to rotation and pairing correlations, we performed the Hartree-Fock-Bogoliubov (HFB) calculations in the current implementation of the code HFODD (v2.17k)[13] for ^{110}Zr, which shows[2] a pronounced tetrahedral minimum at $I = 0$. In our calculations we employed the SLy4[14] Skyrme force in the particle-hole channel and the density-dependent delta interaction in the particle-particle channel, whose intensity was fitted so as to reproduce the trend of experimental pairing gaps in the neutron-rich Zr isotopes, see Ref.[4] for details.

The calculations were performed for two families of rotation axes: (i) the axis passing through the middle of the edge of the tetrahedron, (ii) the axis passing through the tip of the tetrahedron. They will be referred hereafter as to edge and tip axes, respectively. These two axes are given a particular attention as they are

symmetry axes of the rotating tetrahedron; a 4-fold axis with inversion (edge) and a 3-fold axis (tip), i.e. these symmetries commute with the cranking hamiltonian.

For the edge and tip axes, we performed the constrained HFB calculations. In the case of the edge axis we requested that the discrete antilinear T-simplexes, S_x^T and S_y^T, be conserved. (We refer to Ref. [13] for a comprehensive review of the discrete symmetries implemented in the code HFODD). This was equivalent to conserving the z-signature symmetry, R_z. They were supplemented by full symmetry-unconstrained calculations, which was the only option available for the rotation about the tip axis. In the latter case, we also used the option to readjust the orientation of the rotation axis self-consistently in the course of the iterations. We did not notice (i) any change in the orientation of $\vec{\omega}$ fixed at the beginning of the iterations, at least up to the first single-particle crossing, (ii) any significative difference in energy between the various orientations of $\vec{\omega}$ and between the symmetry-constrained and symmetry-unconstrained case. This confirms a spherical-like behaviour of the rotating tetrahedron as claimed in Ref.[5].
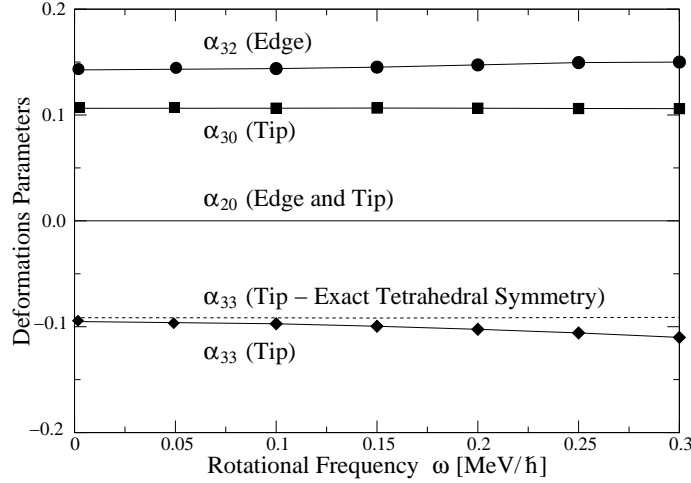

Fig. 1. Evolution of the deformation parameters in the tetrahedral minimum of ^{110}Zr as function of the rotational frequency ω for the edge- (circles) and tip-axes (squares and diamonds). For both classes of axes, the quadrupole deformation remains equal to 0. For the tip-axis, the theoretical value of α_{33} in the exact T_d^D limit is marked as the dashed line.

With the standard choice of the spherical harmonics for the calculation of the multipole moments, the edge-axis coincides with the z-axis of the body-fixed frame[6]. The deformations $\alpha_{\lambda\mu}$ extracted from the multipole moments can be plotted directly, cf. Fig. 1. The curve with plain circles shows the "tetrahedral" deformation α_{32}. It is worth noticing that it is nearly constant as function of ω, thus proving the resilience of the tetrahedral symmetry to the rotation.

In the case of the tip-axis, we chose to first perform a rotation of the nucleus at rest by the Euler angles (α, β, γ) in such a way that the new tip-axis coincides

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with the z-axis of the reference frame. This corresponds to an angle β such that $\cos^2 \beta = 1/3$, and $\alpha = \gamma = 0$. The multipole moments $Q_{\lambda\mu}$ must be transformed according to the general relation:

$$Q'_{\lambda\mu} = \sum_{\mu'} D^{*\lambda}_{\mu\mu'}(\alpha, \beta, \gamma)Q_{\lambda\mu} \tag{1}$$

where the $D^{*\lambda}_{\mu\mu'}(\alpha, \beta, \gamma)$ are the Wigner matrices. In our case, the initial deformation was characterized by $Q_{32} \neq 0$ and all other moments were null. After the rotation, only Q_{30} and Q_{33} are different from zero, and they are related by: $Q_{30}/Q_{33} = -\sqrt{10}/2$. We report in Fig. 1 the equivalent α_{30} and α_{33} deformations as function of the rotational frequency, together with the value of α_{33} in the exact symmetry limit. The little deviation from the exact symmetry case seems to increase slightly with ω although remaining very small.

In conclusion, we have investigated the rotation of tetrahedral nuclei using a fully-microscopic 3-dimensional cranking model. The results of self-consistent Skyrme Hartree-Fock Bogoliubov calculations show that the tetrahedral minimum remains as function of the rotational frequency and that practically no quadrupole polarization remains. The latter observation implies strongly hindered stretched-$E2$ transitions for the eventual rotational bands.

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